Assignment 1

Coverage: 15.1 in Text.

Exercises: 15.1. No 7, 9, 11, 16, 18, 20, 25, 27, 32, 34.

Submit no. 20, 32, and 34 by Sept 14.

Supplementary Problems

1. Consider the function $\varphi(x) = x^{-a}$ where a is positive for $x \in (0, 1]$ and set $\varphi(0) = 1$ so that φ is a well-defined function on $[0, 1]$. Show that φ is not integrable on $[0, 1]$. This is the simplest example of an unbounded function. Suggestion: You could use proof by contradiction. Assume it is integrable and then draw a contradiction.

2. Consider the function H in $\mathbb{R}^2$ defined by $H(x, y) = 1$ whenever x, y are rational numbers and equals to 0 otherwise. Show that H is not integrable in any rectangle.

3. Let $f = f(x, y)$ be a bounded function defined in $R = [0, 1] \times [0, 1]$ which is 0 everywhere except at a point $(1/2, 1/2)$. Show that f is integrable in R with integral equal to 0.

4. Let $g = g(x, y)$ be a bounded function defined in $R = [0, 1] \times [0, 1]$ which is 0 everywhere except along the line $x = 1/2$. Show that f is integrable in R with integral equal to 0.